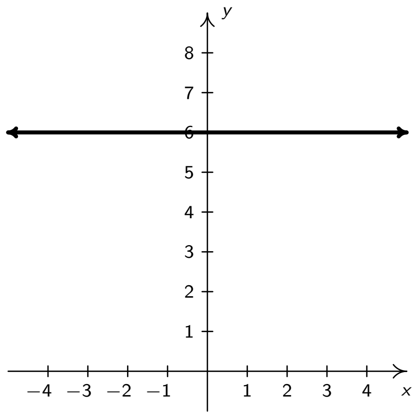
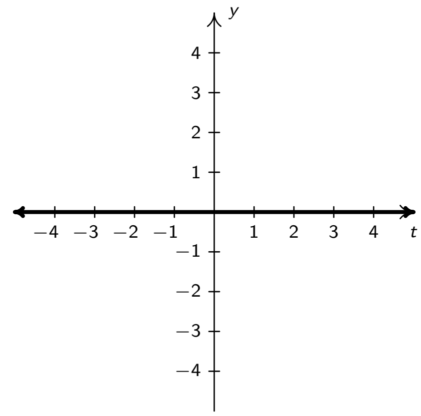
MATH 1650 CONSTANT AND LINEAR FUNCTIONS

EXAMPLE: Graph the following functions.

- $f(x) = 6$

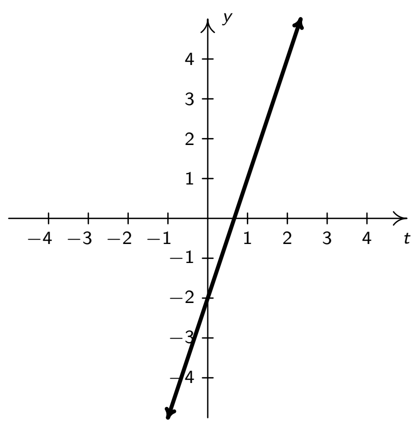


- $g(t) = 0$

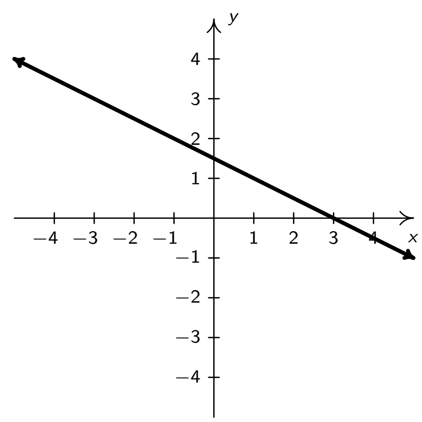


EXAMPLE: Graph the following functions.

- $F(x) = 3x - 2$



- $G(x) = \dfrac{3-x}{2}$



EXAMPLE: The function $T(t) = -2t + 10$ gives the temperature (in °F) on a particular day t hours after 4 PM.

- $T(0) = -2(0) + 10 = 10$. This means at 4 PM (that is, 0 hours after 4 PM), the temperature is 10°F.

- The slope of $T(t)$ is $-2 = \dfrac{-2}{1} = \dfrac{-2^\circ\text{F}}{1\,\text{hour}}$. In other words, the temperature is dropping by 2°F every hour.

- To find when the temperature reaches 0°F, we need to solve $T(t) = 0$. Solving $-2t + 10 = 0$ gives $t = 5$. Hence, at 9 PM (5 hours after 4 PM), the temperature will be 0°F.

EXAMPLE: Suppose a linear function f satisfies $f(-1) = 2$ and $f(3) = 0$.

- Find the slope of the linear function: $m = \dfrac{\Delta[f(x)]}{\Delta x} = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{f(3) - f(-1)}{3 - (-1)} = \dfrac{0 - 2}{4} = -\dfrac{1}{2}$.

- Use the point-slope formula of a linear function to find a formula for $f(x)$:
  - Using $x_0 = -1$ and $f(x_0) = f(-1) = 2$:

    $$f(x) = m(x - x_0) + f(x_0) = -\frac{1}{2}(x - (-1)) + 2 = \ldots = -\frac{1}{2}x + \frac{3}{2}$$

  - Using $x_0 = 3$ and $f(x_0) = f(3) = 0$:

    $$f(x) = m(x - x_0) + f(x_0) = -\frac{1}{2}(x - 3) + 0 = \ldots = -\frac{1}{2}x + \frac{3}{2}$$

  - It shouldn't matter which point we choose to use to determine the linear function because both points are on the same line, so we should always get the same answer.

PIECEWISE-DEFINED FUNCTIONS: Consider the function: $f(x) = \begin{cases} 6 - x & \text{if} \quad x \le 2 \\ 2x - 1 & \text{if} \quad x > 2 \end{cases}$

Notice we have **two** formulas for $f(x)$ depending on what the input is:

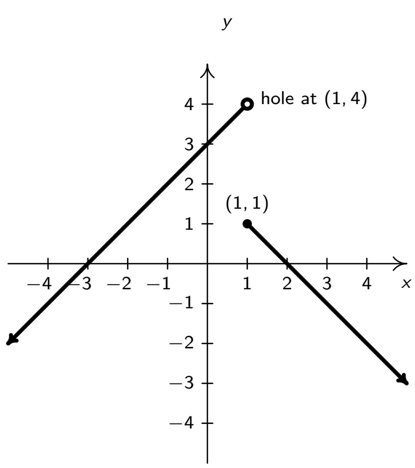
- if $x \le 2$, then we use the formula $f(x) = 6 - x$.
- if $x > 2$, then we use the formula $f(x) = 2x - 1$.

Find the following function values:

- $f(0) = 6 - 0 = 6$
- $f(1) = 6 - 1 = 5$
- $f(2) = 6 - 2 = 4$
- $f(2.1) = 2(2.1) - 1 = 3.2$
- $f(3) = 2(3) - 1 = 5$
- $f(4) = 2(4) - 1 = 7$

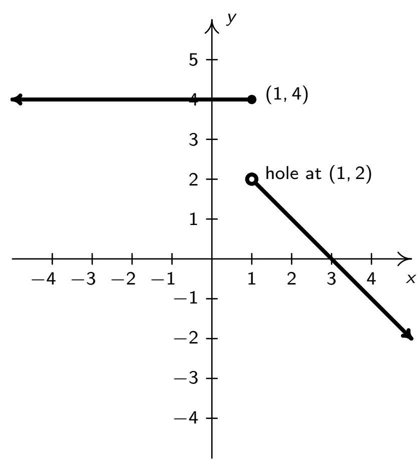
EXAMPLE:

The graph of: $f(x) = \begin{cases} x + 3 & \text{if} \quad x < 1 \\ 2 - x & \text{if} \quad x \ge 1 \end{cases}$



$y = f(x)$

$C(x) = \begin{cases} 4 & \text{if} \quad x \le 1 \\ 3 - x & \text{if} \quad x > 1 \end{cases}$



$y = C(x)$

MATH 1650 APPLICATIONS OF LINEAR FUNCTIONS

1. The cost C (in dollars) to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$.

   (a) $C(0) = 2(0) + 26 = 26$. This means it costs \$26 to produce 0 shirts.

   NOTE: This is sometimes called the **fixed** or **start-up** cost.

   (b) To see how many shirts can be made on a budget of \$117, we solve $C(x) = 117$.

   Substituting $C(x) = 2x + 26$, we get $2x + 26 = 117$ which gives $x = 45.5$.

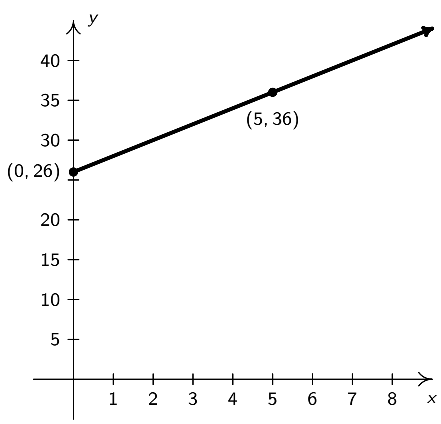
   Since we can't make half a shirt, we either need to make 45 shirts or 46 shirts.

   The cost to make 45 shirts is $C(45) = 2(45) + 26 = 116$, or \$116, which is under budget.

   The cost to make 46 shirts is $C(46) = 2(46) + 26 = 118$ or \$118, which is over budget.

   So on a budget of \$117, we can make 45 shirts.

   (c) Graph $y = C(x)$. **NOTE:** Since x is restricted to $x \geq 0$, the graph starts at the point $(0, 26)$.



$y = C(x)$

   (d) The slope is $2 = \dfrac{2}{1} = \dfrac{\$2}{1\,\text{shirt}}$ which means that it costs an additional \$2 to make 1 additional shirt.

   (e) C is increasing which means the cost continues to rise as more shirts are made.

2. The price p (in dollars per shirt) that is charged for each "I'd rather be a Sasquatch" T-shirt is a function of how many shirts are sold, x.

   (a) Find the domain and range of p.

   domain: $[0, 15]$

   range: $[0, 30]$

   (b) p is decreasing which means to sell more shirts, we must lower the price per shirt.

   (c) To find the slope, we use the two points $(0, 30)$ and $(15, 0)$:

$$m = \frac{\Delta[p(x)]}{\Delta x} = \frac{0-30}{15-0} = -2 = \frac{-\$2}{1 \text{ shirt}}$$

   In other words, for each drop in price of \$2, we'll sell an additional shirt.

   (d) Using the point-slope formula: $p(x) = m(x - x_0) + p(x_0)$ with $x_0 = 0$, we get:

$$p(x) = -2(x-0) + 30 = -2x + 30, \quad \text{for } 0 \leq x \leq 15.$$

   (e) To solve $p(x) = 12$, we substitute $p(x) = -2x + 30$ solve: $-2x + 30 = 12$. We find $x = 9$.

   This means if we set the price at \$12 per shirt, we'll sell 9 shirts.

3. At 8 AM, it is 45° F. At noon, it is 53°F.

   (a) Find a linear function which models the temperature, $T(t)$ as a function of the number of hours after 6 AM, t.

   At 8 AM it's 45° F translates to $T(2) = 45$ since 8 AM is 2 hours after 6 AM.

   At noon, it is 53°F translates to $T(6) = 53$ since Noon is 6 hours after 6 AM.

   To find the slope of the linear function, $m = \dfrac{\Delta[T(t)]}{\Delta t} = \dfrac{T(6) - T(2)}{6-2} = \dfrac{53-45}{4} = \dfrac{8}{4} = 2.$

   The point-slope form of $T(t)$ is: $T(t) = m(t - t_0) + T(t_0)$.

   Choosing $t_0 = 2$, $T(t_0) = T(2) = 45$, we get $T(t) = 2(t-2) + 45 = 2t + 41$.

   Since t represents the number of hours **after** 6 AM, we want $t \geq 0$. Our final answer is:

$$T(t) = 2t + 41, \quad \text{for } t \geq 0$$

   (b) The slope is $2 = \dfrac{2°\text{F}}{1 \text{ hour}}$ which means that it is getting warmer as the day goes on at a rate of 2° F per hour.

   (c) The time 1 PM is 7 hours after Noon. Since $T(7) = 2(7) + 41 = 55$, it will be 55°F at 1 PM.

4. We are given the cost is a constant \$10 if the usage is up to and including 2 Gb, so $C(G) = 10$ if $G \leq 2$. Since we can't use a negative amount of data, $G \geq 0$. So putting all this together, we have $C(G) = 10$ for $0 \leq G \leq 2$.

   We are told the overage fee is \$15 per Gb, so this means if $G > 2$:,

$$C(G) = \text{initial \$10 plus 15 times the amount of Gb } \textbf{over } 2 = 10 + 15(G-2) = 15G - 20$$

   Putting together both cases, we have:

$$C(G) = \begin{cases} 10 & \text{if} \quad 0 \leq G \leq 2 \\ 15G - 20 & \text{if} \quad G > 2 \end{cases}$$

USING SLOPE TO ANALYZE NON-LINEAR FUNCTIONS: AVERAGE RATE OF CHANGE

EXAMPLE: The formula $s(t) = -5t^2 + 100t$ for $0 \le t \le 20$ gives the height, $s(t)$, measured in feet, of a model rocket above the Moon's surface as a function of the time after lift-off, t, in seconds.

- Find and interpret $s(0)$, and $s(10)$, and $s(20)$.

  $s(0) = -5(0)^2 + 100(0) = 0$; the rocket is launched from the Moon's surface.

  $s(10) = -5(10)^2 + 100(10) = 500$; after 10 seconds, the rocket is 500 feet off the surface of the Moon.

  $s(20) = -5(20)^2 + 100(20) = 0$; after 20 seconds, the rocket returns to the surface of the Moon.

- Find the average rate of change of s over the intervals $[0, 10]$ and $[10, 20]$.

  $$ARC_{[0,10]} = \frac{\Delta[s(t)]}{\Delta t} = \frac{s(10) - s(0)}{10 - 0} = \frac{500 - 0}{10} = 50 = \frac{50\,\text{feet}}{1\,\text{second}}.$$

  For the first 10 seconds of its flight, the rocket travels **upwards** (since the ARC is **positive**) at an average speed of 50 feet per second.

  $$ARC_{[10,20]} = \frac{\Delta[s(t)]}{\Delta t} = \frac{s(20) - s(10)}{20 - 10} = \frac{0 - 500}{10} = -50 = \frac{-50\,\text{feet}}{1\,\text{second}}.$$

  For the last 10 seconds of its flight, the rocket travels **downwards** (since the ARC is **negative**) at an average speed of 50 feet per second.